Wing-Slipstream Interaction with Mach Number Nonuniformity

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Theme

A NUMERICAL method for obtaining the wing aerodynamic characteristics with slipstream interaction and Mach number nonuniformity (i.e., with different slipstream and freestream Mach numbers) is presented. The method is based on the quasi vortex-lattice method[1] and a two-vortex-sheet representation of the slipstream.

Recent efforts in this area has been based on either the Weissinger's lifting line method or its variants[2] in incompressible flow so that the wing pressure and the spanwise induced drag distributions are not predicted. Even though Shollenberger[3] has formulated a nonlinear method based on the vortex lattice method, it is not applicable to problems with Mach number nonuniformity. The present method is formulated to remove these limitations within the context of the linear potential flow theory. Results are compared with those given by other theories and available experimental data. The effect of Mach number nonuniformity is explained.

Contents

The nondimensional perturbed velocity potentials inside and outside the slipstream (with subscripts s and o, respectively) are assumed to satisfy the Prandtl-Glauert equations with constant Mach numbers M_s and M_o, respectively. The shape of the slipstream is assumed circular and unchanged throughout. The mean slipstream velocity remains uniform everywhere. The effects of fuselage and nacelles are not included. Assuming that the perturbation velocities are small relative to the freestream and mean slipstream velocities, the conditions of flow tangency and continuity of static pressures on the slipstream surface, respectively, lead to the following linearized slipstream boundary conditions:

$$\frac{\partial \phi_s}{\partial n} = \frac{\partial \phi_o}{\partial n} + \frac{U\cdot n(1-\mu')}{U\cdot e_s} \tag{1}$$

$$\frac{\partial \phi_s}{\partial s} = T(\mu')^2 \frac{\partial \phi_o}{\partial s} \tag{2}$$

The linearized wing flow tangency conditions are

$$\frac{\partial \phi_o}{\partial z} = \frac{U\cdot i}{U\cdot e_s}\frac{\partial z_c}{\partial x} - \frac{U\cdot k}{U\cdot e_s} \tag{3}$$

in the outer flow and

$$\frac{\partial \phi_s}{\partial z} = \frac{V_s\cdot i}{V_s\cdot e_s}\frac{\partial z_c}{\partial x} - \frac{(U\cdot e_z)\ (e_z\cdot k)}{V_s\cdot e_s} - \frac{V_s\cdot k}{V_s\cdot e_s} \tag{4}$$

inside the slipstream. In Eqs. (1-4), e_s and e_z are unit vectors along and normal to the slipstream axis, respectively, and (s, n) are the coordinates along and normal to the slipstream surface while (i, k) are the usual Cartesian unit vectors. $T=\rho_o/\rho_s$, U=outer flow velocity vector, V_s=slipstream velocity vector and $\mu' = U\cdot e_s / V_s \cdot e_s$. Slipstream rotation can

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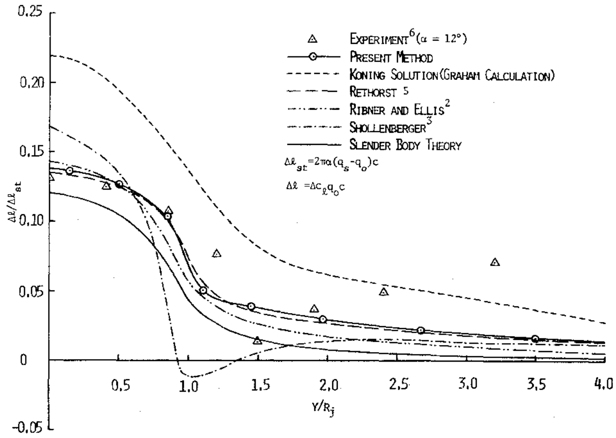

Fig. 1 Comparison of span loadings of a centered-jet configuration predicted by existing theories. $D/c = 0.6$ and $\mu = 0.735$.

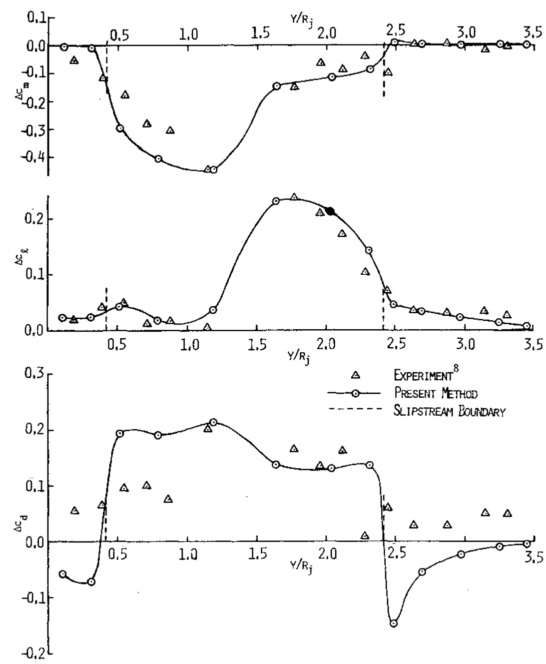

Fig. 2 Comparison of predicted Δc_ℓ, Δc_d and Δc_m with experiments for a rectangular wing of $A = 4.705$ at $\alpha = 10°$. $D/c = 4/3$, $\mu = 0.44721$, $b/2 = 3.529$, $\delta_f = 0°$ and thrust coefficient = 0.8. Moment center at $c/4$.

Table 1 Comparison of predicted aerodynamic characteristics with experiments for wings with slipstream interaction $\alpha = 10°$, $M_o = M_s = 0$

	$A=4.705$, $\delta_f=0$		$A=3.22$, $\delta_f=15°$	
	Present	Experiment[9]	Present	Experiment[9]
ΔC_L	0.771	0.755	1.414	1.43
ΔC_D	0.0752	0.1	0.3044	0.39
ΔC_m	−0.1444	−0.14	−0.371	−0.425

be accounted for through the last term of Eq. (4). For numerical convenience, the preceding problem is solved by setting $\phi_o = \phi_{wo}(M_o) + \psi_o(M_o)$ and $\phi_s = \phi_{ws}(M_s) + \psi_s(M_s)$, where $\phi_{wo}(M_o)$ and $\phi_{ws}(M_s)$ are the wing-alone velocity potentials at the indicated Mach numbers and ψ_o and ψ_s are the additional velocity potentials due to interaction. If follows that in the absence of slipstream, the additional perturbations will automatically vanish in the present numerical scheme. As a result of the Mach number nonuniformity and normal-velocity jump indicated in Eq. (1), two vortex sheets are introduced on the slipstream surface, one to account for the perturbations in the outer flow and the other for those inside the slipstream. The circular slipstream is then approximated by an inscribed polygon, with 8 vortex strips covering the boundary for the following numerical results. All streamwise vortex integrals for the induced velocities are reduced to finite sums and Eqs. (1-4) are satisfied at control points according to the Quasi VLM.[1] The resulting algebraic equations are then solved for the unknown vortex strengths by Purcell's method[4] to reduce the computer memory requirement.

Application of the method gave the following results. 1) For a rectangular wing of aspect ratio $A = 5.25$ with a centered jet of $\mu = V_o / V_s = 0.735$, the present results agree well with Rethorst[5] (see Fig. 1). The disagreement of computed results with experiments[6] outside the slipstream has been attributed to the fan enclosure upstream.[5] 2) The lifting-line theory[7] predicts the span loading of an elliptic wing of $A = 6.5$ inside the slipstream about 10% highter than that given by the present method. 3) Incorporating the experimental slipstream rotation in the term $\boldsymbol{V}_s \cdot \boldsymbol{k}$ in Eq. (4), good agreement in total lift increments with experiments[8] is obtained, while both the slender body theory and Ribner and Ellis[2] tend to underpredict the lift, as is also evident from Fig. 1. 4) In comparison with Nishimura's experiments,[9] it was assumed that Brenckmann's measured slipstream rotational angles[8] are applicable if the quantity, $J(1-\mu^2)/\mu$, obtained from the inviscid propeller theory, is the same, where J is the propeller advance ratio. Reasonably good agreement in spanwise distribution of Δc_ℓ, Δc_d and Δc_m has been obtained, with better accuracy in Δc_ℓ and less in Δc_d predictions, where Δ denotes the incremental values over the wing alone case (see Fig. 2). The overall aerodynamic characteristics are compared in Table 1. That the predicted lift increment is lower with full-span flap deflection may be due to the neglect of effects of slipstream deflection due to flap and the higher predicted wing-alone C_L than the experimental value because of flow separation at high α and flap angle (δ_f). 5) It was found that nonuniformity in Mach numbers (with higher slipstream Mach numbers) increased the loading only slightly at $\mu = -0.44721$ and to a magnitude much less than that which would have been predicted if the Prandtl-Glauert transformation was applied locally. This is due to the compensating effects of reflected and diffracted disturbances which are both increased with increase in slipstream Mach numbers.[10]

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